UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the October/November 2011 question paper

for the guidance of teachers

4037 ADDITIONAL MATHEMATICS

4037/12 Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

OW –1,2 This is deducted from A or B marks when essential working is omitted.

PA –1 This is deducted from A or B marks in the case of premature approximation.

S –1 Occasionally used for persistent slackness – usually discussed at a meeting.

EX –1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

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1 $\dfrac{1}{\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}}$	M1	M1 for adding fractions in terms of sin/cos/tan/cot correctly
$=\dfrac{\sin\theta\cos\theta}{\sin^2\theta+\cos^2\theta}$	M1	M1 for use of correct identity
$=\sin\theta\cos\theta$	A1 **[3]**	A1 for correct solution only
OR $\dfrac{\tan\theta}{\tan^2\theta+1}$ or $\dfrac{\cot\theta}{\cot^2\theta+1}$	M1	M1 for adding fractions in terms of tan/cot correctly
$=\dfrac{\tan\theta}{\sec^2\theta}$ or $\dfrac{\cot\theta}{\text{cosec}^2\theta}$	M1	M1 for use of correct identity
$=\sin\theta\cos\theta$	A1	A1 for correct solution only
2 $(2y+1)^2+y^2=29$ (or $5x^2-2x-115=0$)	M1	M1 for attempt to get an equation in terms of one variable only
leading to $5y^2+4y-28=0$	DM1	DM1 for obtaining a 3 term quadratic equation
(or $x^2+\left(\dfrac{x-1}{2}\right)^2=29$)	DM1	DM1 for attempt to solve quadratic equation
$x=-\dfrac{23}{5}, y=-\dfrac{14}{5}$ and	A1	A1 for a pair of values
$x=5, y=2$	A1	
(5, 2) spotted gets B1	**[5]**	

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3 **(i)** $\dfrac{1}{\log_2 x}$ or $\dfrac{\log_2 2}{\log_2 x}$	B1	
(ii) $u^2 - 3u + 2 = 0$ $(u-1)(u-2) = 0$	M1	M1 for a correct attempt to obtain and solve a quadratic equation in terms of u or $\log_2 x$
$u = 1,\ 2$	A1	A1 for $u = 1,\ 2$
	M1	M1 for attempt to solve an equation of the form $\log_2 x = k$ leading to $x = 2^k$
$x = 2$ and $x = 4$	A1 **[5]**	A1 for both
4 When $x = 2$, $y = 9$	B1	B1 for $y = 9$
$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2}{3}.6x.\left(3x^2+15\right)^{-\frac{1}{3}}$	B1, B1	B1 for $\dfrac{2}{3}.6x$, B1 for $\left(3x^2+15\right)^{-\frac{1}{3}}$
when $x = 2$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{8}{3}$		
$\therefore$ grad of normal = $-\dfrac{3}{8}$	M1	M1 for use of $m_1 m_2 = -1$
normal : $y - 9 = -\dfrac{3}{8}(x-2)$	M1	M1 for attempt to find equation of normal, must be using gradient of a perpendicular line
$8y + 3x = 78$	A1 **[6]**	A1 allow unsimplified
5 **(i)** $y^2 = m2^x + c$	M1	M1 for use of straight line equation as given
$81 = 80 + c$ $c = 1$	M1	M1 for use of (16, 81)
$y^2 = 5\left(2^x\right) + 1$	A1	A1 – do not allow if subsequent incorrect work is seen
(ii) $36 = 5\left(2^x\right) + 1$	M1	M1 substitution of $y = 6$ into their equation in terms of y^2 and 2^x
leading to $7 = 2^x$	DM1	DM1 for correct solution of equation of the form $2^x = k$
$x = 2.81$	A1 **[6]**	

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6 **(i)** $243+405x+270x^2+90x^3+15x^4+x^5$ $243-405x+270x^2-90x^3+15x^4-x^5$ $486+540x^2+30x^4$	 B1 B1 B1 B1	Assuming correct terms in x B1 for 486 B1 for 540 B1 for 30 B1 for all terms correct and no extra terms
(ii) $30y^2+540y-600=0$	M1 DM1	M1 for attempt to obtain a 3 term quadratic equation. DM1 for correct attempt at solution of quadratic
$y=1.05$	A1	A1 need both solutions
leading to $x=\pm1.02$	A1 **[8]**	A1 need both solutions
7 **(i)** $16x^{-\frac{1}{2}}-8+x^{\frac{1}{2}}$	B1, B1, B1	B1 for each correct term
(ii) $y=32x^{\frac{1}{2}}-8x+\frac{2}{3}x^{\frac{3}{2}}(+c)$	M1 A2, 1, 0	M1 for attempt to integrate a 3 term expression –1 for each error
When $x = 9$ and $y = 30$, $c = -12$	M1 A1 **[8]**	M1 for attempt to find c, must have attempted integration A1 for $c = -12$
8 **(i)** $M(2, -1)$	B1	Allow in **(ii)**
Grad $AB = \frac{8}{6}$, $\perp$ grad $= -0.75$ CD: $y+1=-0.75(x-2)$	M1 DM1 A1	M1 for attempt to find gradient of perpendicular DM1 for straight line equation using M
(ii) $C(-2, 2)$ $D(10, -7)$	B1 B1, B1	B1 for y coordinates of C can be awarded in **(iii)** B1 for each of the coordinates for D
(iii) Area $= \frac{1}{2}\sqrt{12^2+9^2}\sqrt{3^2+4^2}$ $= 37.5$ Or $\frac{1}{2}\begin{vmatrix} -2 & 10 & -1 & -2 \\ 2 & -7 & -5 & 2 \end{vmatrix} = 37.5$	M1 A1 **[9]**	M1 for attempt at area

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9 **(i)** $\frac{dy}{dx} = x.4\cos 4x + \sin 4x$	M1 B1, A1	M1 for differentiation of a product B1 for $4\cos 4x$, A1 all else correct
(ii) $I = \left(\frac{1}{4}\right)\left[x\sin 4x - \int \sin 4x \, dx\right]$	DM1	DM1 for realising integration is form reverse process of **(i)** – do not need $\left(\frac{1}{4}\right)$ until last A1
$= \left(\frac{1}{4}\right)\left[x\sin 4x - \left(-\frac{1}{4}\cos 4x\right)\right]$	A1, A1 B1	A1 for $x\sin 4x$, A1 for $\int \sin 4x \, dx$ B1 for $-\frac{1}{4}\cos 4x$
For definite integral $\left(\frac{1}{4}\right)\left[x\sin 4x - \left(-\frac{1}{4}\cos 4x\right)\right]_0^{\frac{\pi}{8}}$	M1	M1 for correct application of limits
$= \frac{\pi}{32} - \frac{1}{16}$, or 0.0357	A1 **[9]**	
10 **(i)** $2\tan^2 x + 2 = 5\tan x + 5$	M1	M1 for use of correct identity
$2\tan^2 x - 5\tan x - 3 = 0$	M1	M1 for solution of 3 term quadratic equation
$\tan x = -\frac{1}{2}, \tan x = 3$	M1	M1 for attempt to solve $\tan x = k$ from a 3 term quadratic equation
$x = 153.4°, 333.4°$ and $71.6°, 251.6°$	A1, A1	A1 for any pair
(ii) $\sin\left(0.5y + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$	M1	M1 for dealing with $\sqrt{2}$ correctly
$0.5y + \frac{\pi}{3} = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}$	M1 M1	M1 for correct order of operations M1 for correct order of operations and attempt to get a solution in the range
leading to $y = \frac{5\pi}{6}, \frac{23\pi}{6}$	A1, A1 **[10]**	Allow decimal equivalents 2.62 and 12.0

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11 EITHER		
(i) $A = 4$	B1	
(ii) $\dfrac{dy}{dx} = e^{-x}\left(-2A\sin 2x + 2B\cos 2x\right) -$	M1 B1	M1 for differentiation of a product B1 for $(-2A\sin 2x + 2B\cos 2x)e^{-x}$
$e^{-x}\left(A\cos 2x + B\sin 2x\right)$	B1	B1 for $-e^{-x}\left(A\cos 2x + B\sin 2x\right)$
when $x = 0$, $6 = 2B - A$, $B = 5$ (verification acceptable)	M1 A1	M1 for substitution to find B
(iii) when $\dfrac{dy}{dx} = 0$,	M1	M1 for their $\dfrac{dy}{dx} = 0$
$e^{-x}\left(p\cos 2x - q\sin 2x\right) = 0$	M1	M1 for attempt to simplify
leading to $\tan 2x = \dfrac{p}{q}$ $\left(= \dfrac{6}{13}\right)$	M1	M1 for attempt to obtain $\tan 2x = \dfrac{p}{q}$
$x = 0.216$	M1 A1 **[11]**	M1 for attempt to solve $\tan 2x = k$
11 OR		
(i) $\dfrac{dy}{dx} = \dfrac{\left(x^2-1\right)\dfrac{2x}{\left(x^2-1\right)} - 2x\ln\left(x^2-1\right)}{\left(x^2-1\right)^2}$	M1 B1	M1 for differentiation of a quotient B1 $\dfrac{2x}{\left(x^2-1\right)}$
Rearranging to get $k = 2$	A1 A1	A1 for all else correct A1 for rearrangement to get $k = 2$
(ii) $\partial y = \dfrac{dy}{dx}p$,	M1	M1 for substitution of $x = \sqrt{5}$ and correct method
leading to $\partial y = -0.108p$	√A1	√A1 on their k
(iii) when $\dfrac{dy}{dx} = 0$, $1 - \ln\left(x^2-1\right) = 0$	M1	M1 for $\dfrac{dy}{dx} = 0$ and attempt to simplify
$\ln\left(x^2-1\right) = 1$	A1	A1 for $\ln\left(x^2-1\right) = 1$
$x^2 - 1 = e$ or 2.72	A1	A1 for $x^2 - 1 = e$ or 2.72
leading to $x = \sqrt{1+e}$	A1	
$y = \dfrac{1}{e}$	A1 **[11]**	

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